The Periodic Sublayer and Heat Transfer to High Prandtl Number Fluids

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Although the periodic or penetration descriptions of the turbulent boundary layer oversimplify this phenomenon by treating a complex statistical process in terms of some mean period, such models often lead to simple, accurate analytical relationships for evaluation of the important transport processes. A principal defect of these descriptions is the poor agreement observed between experiment and prediction for the case of heat or mass transfer with high Prandtl or Schmidt numbers. Katesbas and Gordon (1974) showed that improved agreement could be obtained by postulating Prandtl number dependency of the minimum sublayer decay thickness $\delta_1{}^+$ used by Meek and Baer (1970). The concept of a Prandtl number dependent minimum penetration thickness is not very mechanistically satisfying, since it is postulated that the growth and decay of the boundary layer are dynamic processes. It is unlikely that such processes would be affected by the fluid Prandtl number or the Schmidt number for the mass transfer process. This deficiency of these formulations, as will be shown later, is the same defect of the early steady state models; the turbulent eddy diffusion in the viscous region of the sublayer was not considered.

Because of the difficulty in formulation of the boundary condition for heat transfer at the position of the minimum decay thickness, earlier calculations for the heat transfer case of Meek and Baer (1973) were made by means of a numerical model which required solution of the temperature field over several growth and decay cycles. An adequate formulation of this difficult boundary condition can be obtained by recognition that in the region near the wall from $0 < y^+ < \delta_1{}^+$, the temperature field is the same at the beginning and the end of each cycle. If a quadratic temperature distribution is assumed in this region, an analytical representation of the developing temperature field can be obtained, and the constants in the assumed profile can be obtained since the temperature gradient at the wall and at $\delta_1{}^+$ are the same at the beginning and end of each cycle. The wall temperature is assumed constant throughout the cycle, which is an adequate approximation for a metallic wall. Although quite complex, a closed form solution for a Stanton number can be obtained which is presented as Equation (1):

$$N'_{ST} = \frac{0.5}{P^+ u_i{}^+ N_{Pr}{}^{1/2}} \left\{ G_o \left[\frac{1}{\zeta} \operatorname{erf} \zeta + \frac{2}{\sqrt{\pi}} e^{-\zeta^2} - 2\zeta \operatorname{erfc} \zeta \right] + 2(1-\sigma) \left[\frac{2e^{-\zeta}}{\sqrt{\pi}} - 2\zeta \operatorname{erfc} \zeta \right] + \frac{4}{3\sqrt{\pi}} (\sigma - G_o) \left[\frac{1}{\zeta^2} - \frac{e^{-\zeta^2}}{\zeta^2} + 2e^{-\zeta^2} - 2\sqrt{\pi}\, \zeta \operatorname{erfc} \zeta \right] \right\} \quad (1)$$

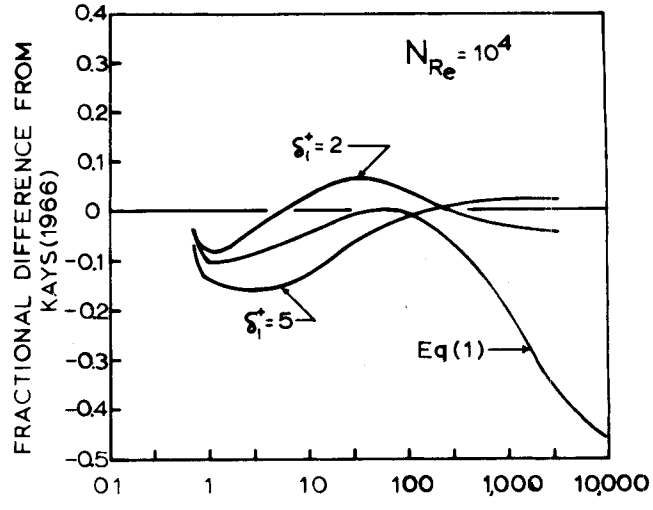

Fig. 1. A comparison of heat transfer coefficient calculated from the periodic sublayer model to Kays (1966) correlation. The values for the curves labeled $\delta_1{}^+ = 2$, 5 include the effect of eddy diffusion in the sublayer region. Values for the Equation (1) line are based on the wall-to-mean fluid temperature difference.

The assumption of a linear temperature profile yields a relationship which gives essentially equivalent results for $1 < N_{Pr} < 200$ and can be obtained from Equation (1), since in this case $G_o = \sigma$. Equation (1) is easily evaluated by use of a computer and has some value for checking the numerical solutions. The Stanton number obtained by use of Equation (1) is in terms of the temperature difference between the wall and the fluid at the position of the maximum growth distance of the laminar sublayer. By use of the logarithmic form of the temperature distribution in the turbulence core, as obtained from mixing length theory, it is possible to use these Stanton numbers to obtain the result conventionally based on the mean stream temperature, as discussed by Meek and Baer (1973).

Figure 1 presents a comparison of the predictions from Equation (1) for a minimum decay thickness of $\delta_1{}^+ = 1.5$ to the accepted correlation of Kays (1966). As in-

dicated above, the predictions of the periodic model become significantly low for fluid Prandtl numbers greater than about 300. This discrepancy is the result of neglect of the transport of energy by eddy motion in the region near the wall. Figure 1 also shows a comparison to Kays (1966) results obtained from the periodic model in which the Deissler (1955) expression for eddy diffusion in the region $0 < y^+ < 5$ was used. In this case, a numerical calculation was required. A value of $\delta_1{}^+ = 5$ was assumed to correspond to the usual steady state value for the edge of the viscous sublayer. In this case agreement between the predictions of the periodic model and a very complete steady state formulation is nearly within the probable uncertainty of all descriptions over the expected range of Prandtl or Schmidt numbers. It should be noted that all parameters of the periodic model are now traceable to prior steady state values; no arbitrary constants are required.

This discussion could end here except that additional information is available. It is possible to measure the root-mean-square value of the wall temperature fluctuations relative to the mean fluid-to-wall temperature difference by use of heat flux gauges mounted on insulating walls. Meek (1968) made such measurements with air and Tetralin as the fluid, and Barlow (1974) considered deionized water and Tetralin. In each investigation, Pyrex tubes were used, and platinum-film resistance thermometers roughly 0.1 mm wide by 2 mm in length in the axial direction were bonded to the surface. For a value of $\delta_1{}^+ = 5$, the calculated wall temperature fluctuations were less than the measured values by at least an order of magnitude for all fluids considered. Significantly better agreement between observation and prediction of the fluctuating wall temperature is obtained if a lower value of $\delta_1{}^+$ is assumed. Table 1 presents a comparison for an assumed value of $\delta_1{}^+ = 2$. The values of the measured temperature fluctuation were slightly adjusted to a common value of the Reynolds number by an empirical fit to sets of data. Because a temperature difference of 50° to 150°C was required to obtain usable signals with air as the fluid, it was necessary to account for the variable air density in the boundary layer. The wall temperature fluctuation data of air were strongly correlated with the wall-to-fluid temperature difference, and it was thus possible to obtain a regression relationship to estimate the fluctuation data at a zero temperature difference. The data in Table 1 were so corrected.

The measurement of the relative wall temperature fluctuations is difficult, and considerable data scatter occurred as is indicated by the large standard deviations of the data. In all cases, the difference between experimental and calculated values of the fluctuations is less than twice the standard deviation of the data, which suggests consistency, but no claim of absolute agreement can be made. An evaluation of the heat transfer predictions for the case that $\delta_1{}^+ = 2$ was made, and these results are presented in the comparison plot Figure 1. In this case, the value of the Deissler constant (his n^2) was reduced from 0.0154 to 0.0125, since such close penetration to the wall region would contribute to the effect of eddy diffusion. The agreement between the results from the steady state correlations and the predictions of the periodic model shown in Figure 1 for $N_{Re} = 10^4$ is typical of the results obtained at higher Reynolds numbers.

CONCLUSIONS

The periodic model of the turbulent boundary layer predicts heat transfer rates in essential agreement with accepted steady state correlations over the practical range of Prandtl and Schmidt numbers when the Deissler (1955) description of the eddy diffusion near the wall is included in the formulation. If an average minimum turbulence penetration distance of 2, in law of the wall units, is assumed, reasonable agreement between the calculated and measured magnitude of the wall temperature fluctuations is also obtained for air, water, or Tetralin as the fluid.

Table 1. Measured and Calculated Values of the Wall Temperature Fluctuations

Fluid	Air	Water	Tetralin
Prandtl No.	0.74	3.0	20.5
Mean N_{Re}	4.5×10^4	3.0×10^4	4.0×10^4
Calculated relative root-mean-square wall temperature	4.5×10^{-4}	7.2×10^{-2}	1.3×10^{-2}
Measured relative root-mean-square wall temperature	6.0×10^{-4}	5.1×10^{-2}	2.4×10^{-2}
Standard deviation	0.9×10^{-4}	1.7×10^{-2}	0.7×10^{-2}
Number of runs	68	52	42

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NOTATION

G_o = ratio of initial qadratic temperature gradient to a linear gradient
$= \dfrac{2}{\sqrt{\pi} D_1} (\zeta e^{-\zeta^2} - \sigma D_4)$

N'_{ST} = modified Stanton Number based on temperature difference from wall to edge of growing sublayer

P = growth and decay period, $P^+ = u_* \sqrt{P/\nu} \cong 18$

u = velocity, $u_* = \sqrt{\tau_w/\rho}$, $u_i{}^+$ = dimensionless velocity at edge of growing sublayer $= u_i/u_*$

y = distance from wall, $y^+ = y u_*/\nu$

Greek Letters

$\delta_1{}^+$ = dimensionless minimum sublayer thickness

ζ = penetration modulus
$= \delta_1{}^+ \sqrt{N_{Pr}}/2P^+$

ν = fluid kinematic viscosity

ρ = fluid density

τ_w = mean shear stress at the wall

σ = ratio of initial wall to fluid temperature difference at $\delta_1{}^+$ to value edge of sublayer
$= (2D_2 \zeta e^{-\zeta^2} - 0.5\sqrt{\pi} D_1 \operatorname{erf} 2\zeta)/(\sqrt{\pi} D_1 D_3 + 2D_2 D_4)$

Additional Constants

$$D_1 = \operatorname{erfc} \zeta - \frac{2}{\sqrt{\pi}\zeta} (e^{-\zeta^2} - 1)$$

$$D_2 = \frac{1}{2\sqrt{\pi}\zeta} (2e^{-\zeta^2} - e^{-4\zeta^4} - 1) + \operatorname{erf} \zeta - \operatorname{erf} 2\zeta$$

$$D_3 = \frac{1}{\sqrt{\pi}\zeta} (e^{-\zeta^2} - 1) - \operatorname{erfc} \zeta - (1/2) \operatorname{erf} 2\zeta$$

$$D_4 = \zeta e^{-\zeta^2} + \frac{1}{\zeta} (e^{-\zeta^2} - 1)$$

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Oxidation of Naphthalene in Packed-Bed Reactor with Catalyst Activity Profile: A Design Scheme for Improved Reactor Stability and Higher Product Yield

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Vapor phase catalytic oxidation of naphthalene to phthalic anhydride using air is usually carried out in packed-bed tubular reactor at 350° to 400°C. Under the same conditions, phthalic anhydride also undergoes oxidation, giving undesirable products. Both the reactions are extremely exothermic.

The heat of reactions is removed by using molten salt as a coolant. However, owing to high exothermicity of the reactions, the temperature rises sharply in the catalyst bed to a maximum (hot spot temperature) and subsequently decreases. To prevent rapid catalyst deterioration, the hot spot temperature must be kept within bounds while a generally high temperature is maintained throughout the remainder of the reactor to give high conversion. In practice this means the reactor must be operated near the unstable region. This causes critical problems because the hot spot is very sensitive to the inlet temperature and inlet naphthalene concentration. Small increases in the latter variables can cause a normally stable reactor to become unstable, leading to runaway and explosion.

Calderbank et al. (1969) and Caldwell and Calderbank (1969) suggested the use of a mechanical mixture of inert and active catalyst pellets to reduce the activity in the front part of the bed in order to control the temperature and thereby increase the stability of the reactor. Apparently, this technique with graded beds of increasing catalytic activity has been used for many years industrially, but little has been written about it.

An alternate approach is to dilute a catalyst with inerts prior to pelleting in order to produce a series of catalysts with similar catalytic characteristics but of reduced activity. Koros and Nowak (1967) have suggested this means of varying the activity of a catalyst. This latter approach, with a diluted catalyst used, has several advantages over the mechanical mixture of inert and active pellets referred to herein as the diluted bed.

The practical problem of nonuniform distribution of catalyst and inert pellets is possible with diluted beds. A distribution having too many active pellets in close proximity in the hot spot region could lead to reactor instability, particularly when a small ratio (8 to 10) of reactor to particle diameter is used as in the case of naphthalene oxidation. This problem will be entirely avoided if diluted catalysts are used. Also, the use of diluted catalysts will make the temperature between the catalyst and the bulk gas lower than the corresponding difference in a diluted bed of catalyst, because the external heat transfer area is increased for the diluted catalyst proportional to the dilution ratio. This is particularly important in the vicinity of the hot spot and leads to a more stable reactor. A comparison of two catalyst beds of the same activity per unit volume of reactor will show that the bed composed of diluted catalyst will have the smaller Thiele parameter than one formed by diluting the bed with inerts; consequently, the selectivity for the reaction system with the intermediate as the desired product will be higher for the diluted catalyst (Wheeler, 1951), particularly in the region of the hot spot. Indeed, Watanabe et al. (1968) oxidized o-xylene to phthalic anhydride for a prolonged period using a reactor with four different activities, product selectivity being about 20% higher than for the reactor with uniform but relatively high activity. The higher selectivity is partly due to the use of catalyst with lower activity and partly due to reduced hot spot temperature. As far as handling the pellets, loading the reactor, and regeneration of catalyst bed are concerned, it seems that there is no advantage of one approach over the other.

It appears, therefore, that reactors with pellets of diluted catalyst offer greater opportunities for increased stability and selectivity than mechanical mixtures of active and inert pellets, and as a consequence attention will be given to how the diluted catalyst activity should be distributed in the bed. It appears that superior performance is associated with catalyst bed having four different activities, a short section of high activity to start the reaction, a longer section of substantially reduced activity in the vicinity of the hot spot zone to induce stability, followed by sections of increasing activity to give high conversion. This will be compared to beds of uniform activity.

KINETICS AND REACTOR MODEL

The reactions involved during naphthalene oxidation can be represented as

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